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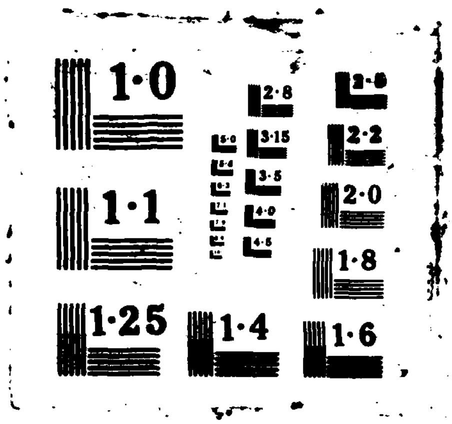
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Let $\mathbb{Z}$ be the set of integers. We denote by m,n etc. the elements of $\mathbb{Z}^2$. Let U be the open unit disc and T the boundary of U in the complex plane $\mathbb{C}$. Let $\mathbb{Z}^2, \mathbb{C}^2, U^2$ and T^2 be the respective calesian product and σ_2 the normalized Lebesgue measure on T^2. For p>0, we denote by $L^p(T^2,\sigma_2)$ the normalized Lebesgue space of the equivalence class of p-integrable functions.

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ON THE VALIDITY OF BEURLING THEOREMS IN POLYDISCS

by

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ON THE VALIDITY OF BEURLING THEOREMS IN POLYDISCS

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Let $\mathbb{Z}$ be the set of integers. We denote by m,n etc. the elements of $\mathbb{Z}$. Let U be the open unit dis .nd T the boundary of U in the complex plane $\mathbb{C}$. Let $\mathbb{Z}^2, \mathbb{C}^2, U^2$ and T^2 be the respective calesian product and σ_2 the normalized Lebesgue measure on T^2. For $p > 0$, we denote by $L^p(T^2, \sigma_2)$ the usual Lebesgue space of the equivalence class of p-integrable functions and $H^p(U^2) = \{f : f : U^2 \to \mathbb{C}$ analytic and $\sup_{0 \le r \le 1} \int_T |f_r(\underline{t})|^p d\sigma_2 < \infty\}$. Here $f_r(\underline{t}) = f(z)$ with $z = r\underline{t}$. Let $\underline{z} = (z_1, z_2) = (r_1 e^{i\theta}, r_2 e^{i\theta_2})$ and $\underline{t} = (e^{i\theta_1, i\theta_2})$, then $P(\underline{z}, \underline{t}) = P_{r_1}(\theta_1 - \theta_1)$. $P_{r_2}(\theta_2 - \theta_2)$ is called Poisson Kernel with

$P_r(\theta) = \dfrac{1 - r^2}{1 - 2r\cos\theta + r^2}$. It is known that for $f \in H^p(U^2)$, $\lim_{r \to 1} f_r(\underline{t}) = f^*(\underline{t})$ exists and is in $L^p(T^2, \sigma_2)$. For $f \in L^p(T^2, \sigma_2)$, let $f^e(z) = \int_{T^2} P(\underline{z}, \underline{t}) f(t) d\sigma_2$, then $f^e \in H^p(U^2)$. In case $p = 2$, $f \in H^2(U^2)$ if f $L^2(T^2, \sigma_2)$ and $f(\underline{t}) = \Sigma_{m=0} \Sigma_{n=0} a_{mn} t_1^m t_2^n$ and $f^e = \Sigma_{m=0} \Sigma_{n=0} a_{mn} z_1^m z_2^n$. Conversely every $f \in H^2(U^2)$ has this form and $f^*(\underline{t}) = \Sigma_{m=0} \Sigma_{n=0} a_{mn} t_1^m t_2^n$. For further information, see [4].

In [4], Rudin gives an example of a shift-invariant subspace of $H^2(T^2)$ which is not of the form $q \cdot H^2$, where q is an inner function. Our purpose here is to characterize invariant subspaces of the form qH^2 in terms of the action of the shifts on it. We note that subspaces of the form qH^2 can be represented as

$$(1) \qquad qH^2 = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \oplus\, V_1^m V_2^n (M)$$

where M equals the span of q in $H^2(T^2)$ and V_i is the multiplication by t_i on $H^2(T^2)$ with $\underline{t} = (t_1, t_2) \in T^2$. It is easy to check that $M = \{qH^2 \ominus V_1(qH^2)\} \cap \{qH^2 \ominus V_2(qH^2)\}$. As V_1 commutes with V_2 (in short, $V_1 \sim V_2$), we get from (1) and Theorem 4.1 of [2] (see also [5]) that V_1 and V_2 are doubly commuting (i.e. $V_1 \sim V_2$, $V_1 \sim V_2^*$). In fact, we have

2. Theorem. An invariant subspace $M \neq \{0\}$ of $H^2(T^2)$ is of the form $q\cdot H^2$ with q inner function if and only if V_1 and V_2 are doubly commuting on M.

Proof: Necessity was proved above. To prove the sufficiency we get, in view of Theorem 4.2 ((c) $\Rightarrow$ (b)) [2],

$$(3) \qquad M = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \oplus V_1^m V_2^n (R_1^\perp \cap R_2^\perp),$$

where $R_i = M \ominus V_i M$, using Theorem 1.2 of [2]. Here we observe that $\bigcap_n V_2^n(R_1^\perp)$, $\bigcap_m V_1^m(R_2^\perp)$ and $\bigcap_{m,n} V_1^m V_2^n(H^2(T^2))$ equal zero. Let $q_1, q_2 \in R_1^\perp \cap R_2^\perp$, then $\int_{T^2} t_1^m t_2^n q_1 \bar{q}_2 d\sigma_2 = 0$ for $\underline{m,n} > 0$. As $V_2(R_1^\perp) \subseteq (R_1^\perp)$ (Lemma 1.3 and Theorem 4.2 of [2]), we get $\int_{T^2} t_2^n q_1 \overline{t_1^m q_2} d\sigma_2 = 0$ for all $n > 0$ and $m > 0$. Since $\overline{t_1^m} = t_1^{-m}$, by the symmetry of the problem we get for $(m,n) \neq (0,0)$

$$\int t_1^m t_2^n q_1 \bar{q}_2 d\sigma_2 = 0.$$

Since $q_1 \bar{q}_2 \in L^1(T^2, \sigma_2)$ we get $q_1 \bar{q}_2 = c_1$ a.e. σ_2. In particular, $|q_1|^2 = c_2$. Hence $R_1^\perp \cap R_2^\perp$ is one dimensional. Also q generating $R_1^\perp \cap R_2^\perp$ is an inner-function. Assume $|q| = 1$ a.e. choosing q of norm 1. Now (3) gives the result.

Let $f \in H^2(T^2)$ and $M_f = \overline{sp}\{V_1^m V_2^n f : m,n \geq 0\}$ then M_f is an invariant subspace.

4. Corollary. $M_f = qH^2(T^2)$ if and only if V_1 and V_2 are doubly commuting on M_f.

Following Helson [1], we say that a function g is H-outer if $M_g = H^2(T^2)$.

5. Corollary. A function $f \in H^2(T^2)$ has the property $f = q\cdot g$ with q inner and g H-outer if and only if V_1 and V_2 doubly commute on M_f.

Proof: By Corollary 4, only if part follows as $M_f = qH^2(T^2)$. To prove the converse we note that by Corollary , $M_f = qH^2(T^2)$ giving $f = q\cdot g$, $g \in H^2(T^2)$. Hence $M_f = q\cdot M_g$ giving g is H-outer.

In ([4], p. 72) a function $f \in H^2(U^2)$ is called outer (we call it R-outer) if $\log|f(z)| = \int_{T^2} \log|f^*| d\sigma_2$. Given a function $f \in H^2(T^2)$, we denote by $f^e \in H^2(U^2)$ given by $\int_{T^2} P(z,t) f(\underline{t}) d\sigma_2$. In this case we note that $(f^e)^* = f$.

It is already known ([4], Theorem 4.4.6) that f is H-outer then f^e is R-outer. From this we get in view of Corollary 4 the following.

6. Corollary. Let g be H-outer then g^e is R-outer and V_1 and V_2 doubly commute on M_g.

We now prove the converse of Corollary 6. Assume now that V_1, V_2 doubly commute on M_f and f^e is R-outer then by Corollaries 5 and 6, the definition of f^e, g^e and the fact that $|q| = 1$ we get $f^e = pg^e$ with $|p| = 1$. Thus we get that the slice function $f^e_w(\lambda) = p_w(\lambda) g^e_w(\lambda)$. Using Lemma 4.4.4(a) of [4] and the uniqueness of outer function ([1], p. 13) we get $p_w(\lambda) = 1$ for all ω and λ giving $p = 1$ i.e., $f = g$. Combining this with Corollary 6 gives

7. *Corollary*. *Let* $f \in H^2(T^2)$ *then* $M_f = H^2(T^2)$ *if and only if* f^e is R-outer and V_1 and V_2 doubly commute.

In view of Theorem 4.2 of [2], we get that Corollary 7 includes Beurling Theorem proved in ([6], Theorem 1.5). Now using essentially classi~~fied~~cal techniques ([3],[2]) one can derive associated results in prediction theory in [6].

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